Investigation of the Link between Physics and POD Modes

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ABSTRACT

The present study aims to clarify the link between vortical structures contained in instantaneous two-dimensional vector fields and their Proper Orthogonal modes. For this purpose, a set of synthetic images is decomposed and analyzed with the aid of a POD algorithm. It is shown that the higher intensity (vector length) of the modes is linked to the ensemble averaged kinetic energy of the corresponding vortices in the vector fields. This energy is correlated to the number of identical vortices in the initial synthetic dataset. The algorithm is also applied on a data set obtained from measurements over a rectangular open cavity thanks to Particle Image Velocimetry (PIV). The vortical structures contained in this last flow are conjectured to be statistically equivalent to the leading POD modes of both the vector and the vorticity fields.

1.0 INTRODUCTION

Modern experimental and/or numerical investigations in fluid dynamics generate a large amount of instantaneous data. Usually post-processing of these data focuses on the Reynolds averaged properties. Nevertheless, it is believed that the instantaneous data still contain valuable information that will, if adequately post-processed, offer a deeper analysis of some mechanisms such as the role of vortex dynamics on turbulence. Partly due to inherent difficulty associated to this last field, special tools are required in order to automatically extract, organize and classify these vortical structures also called Coherent Structures (CS) **[13]**. These events are felt to be responsible for the turbulent mixing, but also for the turbulent transfer in general (heat, mass, momentum) and for the aero-acoustical noise. The identification and extraction of coherent structures is nowadays one of the main goals and mostly investigated domain in turbulence research. One way leading to an extraction of statistical coherent structures is the Proper Orthogonal Decomposition method (POD) **[1]**. Since the pioneer work of Lumley **[7]**, this method became a fashionable tool to extract coherent structures (**[1]**,**[3]**,**[4]**,**[5]**,**[6]**,**[7]**,**[11]**). In this procedure a large amount of flow field realizations serve to extract the most representative structures (on an energetic view point) having a major influence both on the mean flow and the turbulence intensity fields. As a result, it is not always reminded that this most representative structure is also a statistical averaged one that is not always directly present in an instantaneous realization. Luckily, this fact has no consequence if POD is used as a way to find the minimal basis functions allowing to obtain a Reduced Order Model of the flow (**[1]** ,**[7]**,**[10]**,**[11]**,**[16]**). In this last method, the Navier-Stokes (NS) equations are projected and solved on a space where the basis functions are the POD leading modes before the NS solution is transformed back in the physical space. In this way some authors (**[8]**,**[9]**,**[10]**) use POD as a space and time filter on their results before to identify flow structures in real time and space.

Out of this last method the direct analysis of POD modes has to be conducted with care as it will be shown hereafter.

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The organisation of the present article is based on five sections. After, the introduction, a second section contains a short theory of POD. The section 3 exemplified the link between vortical structures and the POD modes through the decomposition of a synthetic vector field while section 4 conjectures the vortical contents of the flow over a cavity based on its modes. Finally, section 5 offers a conclusion together with the future work to appear in this project.

2.0 BASICS OF POD

Proper Orthogonal Decomposition (POD) is a modal decomposition of an arbitrary variable into a series of infinite number of modes. The modes in case of POD are searched with the constraint of being all orthogonal to each other. POD modes are dependent on the phenomena to be decomposed. The decomposition is demonstrated on the velocity field:

$$\vec{u}(\vec{r},t) \cong \sum_{k=1}^{K} a^{(k)}(t)\cdot\vec{\phi}^{(k)}(\vec{r}) \qquad (1)$$

Where $\vec{u}(\vec{r},t)$ is the velocity vector field; $a^{(k)}(t)$ is the temporal scalar coefficient of mode *k* in the time instant *t* and $\vec{\phi}^{(k)}(\vec{r})$ is the spatial mode. Without any other constraint, this decomposition would not be unique. The first restriction is to look for a minimum set of modes which is also a complete basis of the space of realizations. This restriction is met if each element of the basis (i.e. mode) is prescribed to be orthogonal to any other element. Using this constraint each $a^{(k)}(t)$ coefficient would depend only on the corresponding $\vec{\phi}^{(k)}(\vec{r})$ mode.

Among the possible orthogonal basis, the second restriction is to look for a normed basis (L^2-norm) that maximizes the decomposed energy on a minimal number of leading modes. It can be proven (**[1]**,**[2]**) that both restrictions are met if the modes are the spatial eigenfunctions of the following Fredholm integral equation:

$$\sum_{j=1}^{n}\int_{D} R_{ij}(\vec{r},\vec{r}\,')\vec{\phi}_j(\vec{r}\,')d\vec{r}\,' = \lambda\vec{\phi}_i(\vec{r}) \qquad (2)$$

where $R_{ij}(\vec{r},\vec{r}\,')$ is the cross-correlation tensor, *n* is the number of vectorial components of $\vec{u}(\vec{r},t)$. In the Sirovich's method **[12]** the computation of the spatial modes from Eq.(2) is carried out using the time realizations of the flow fields and the cross-correlation degenerates into an auto-correlation. Hereafter, the same Snapshot POD is used. The aim of the present article is not to present a complete derivation of the spatial modes with all the mathematical evidences, the interested reader should refer to **[1][2]** for a full description.

When obtained, the POD modes contain flow-field-like structures as it was assumed in the original method of Sirovich **[12]**. Due to the temporal correlation and integration over space during POD procedure, the obtained structures are arguable to be physical. As it will be shown in the next section, the autocorrelation method merges the temporal information to create a spatial structure statistically present in time. This fact present no trouble if POD is used with filtering purposes **[7][8][10]**, but may be at risk when POD modes are analyzed as realistic flow structures **[6]**,**[3]**. However, recently the method is used for post-processing **[15]**,**[17]**. In the next section, POD modes obtained from a controlled set of flow realizations is used to show the danger of a direct mode analysis.

3.0 TEST OF POD ON SYNTHETIC FLOW FIELDS

In the present section, POD is carried out on datasets which consist of “artificial” flow fields. Each element of the synthetic flow fields may display a uniform flow or an isolated vortex embedded in a uniform flow. The final dataset will contain a total number of flow fields equal to 100. It has been checked that the following results are quite insensitive to the addition of a random noise. Two “synthetic experiments” are conducted. In the first one, the isolated vortex may be located at three different X locations (left, centre, right) with the distance between each location being higher than the vortex diameter. In the second dataset, a high number of X positions is chosen resulting in a distance between each position reduced to 5% of the vortex diameter.

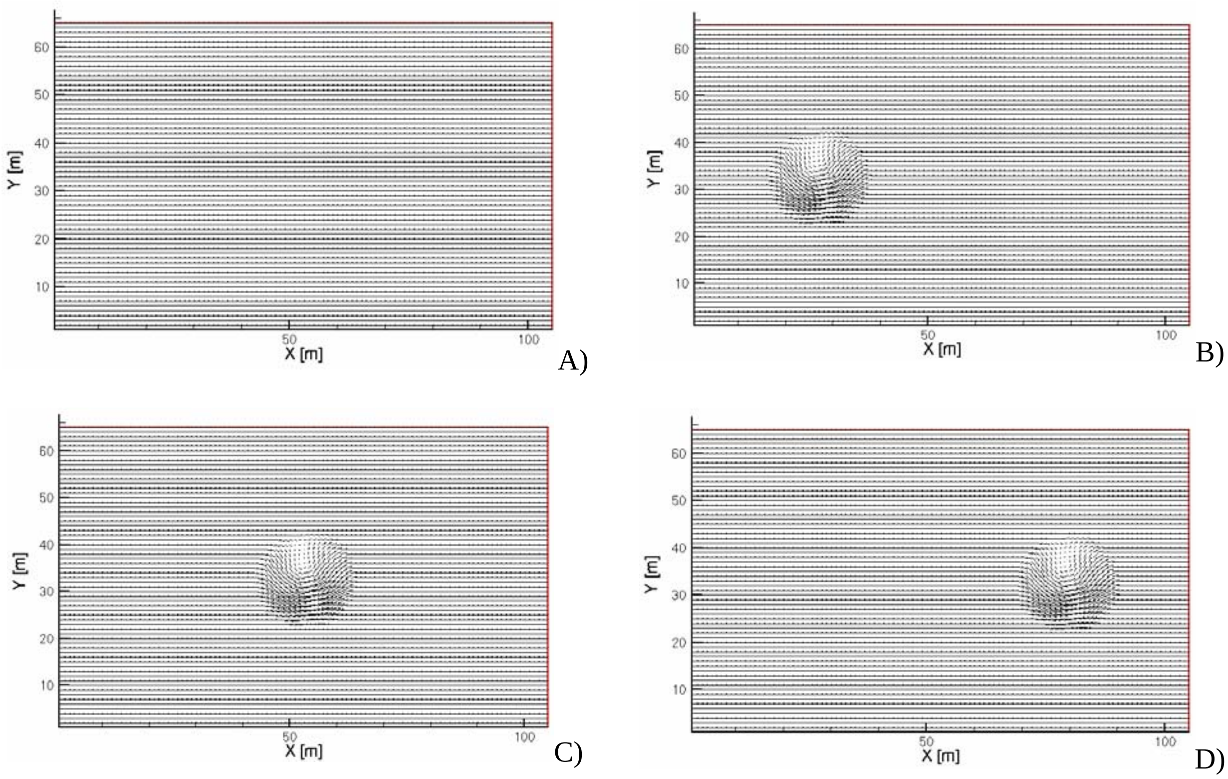

Figure 1: Typical elements of the synthetic dataset. A) Uniform flow fields (50%). B) One vortex on the left side (30%). C) One vortex in the centre (15%). D) One vortex on the right side (5%).

3.1 Non-overlapping vortex positions

This dataset is composed by four types of flow fields presented in **Figure 1**. The total number of realization in the set is equal to 100. With 50% of the realizations are a uniform field (**Figure 1-A**), 30% of the realizations are an isolated vortex embedded in the right part of a uniform field (**Figure 1-B**), 15% of the realizations are an isolated vortex embedded in the centre part of a uniform field (**Figure 1-C**) and the complement of the dataset are an isolated vortex embedded in the right part of a uniform field (**Figure 1-D**). The POD modes of the obtained synthetic dataset can be seen in **Figure 2**. Mode 1 is in close agreement with the ensemble average. In modes 2, 3 and 4, the isolated vortex is retrieved with its correct original geometry in all three positions. It is clearly noticeable that if the spatial characteristics are preserved, the temporal ones are “merged” (two or three footprint of the vortices may appear on a single mode).

It is also observed that the intensity of each vortex pattern (vector lengths in the present case) is proportional to the number of vortices at this position in the decomposed dataset. When velocity components are decomposed (present case), the vector length (intensity) also coincides with the local averaged kinetic energy.

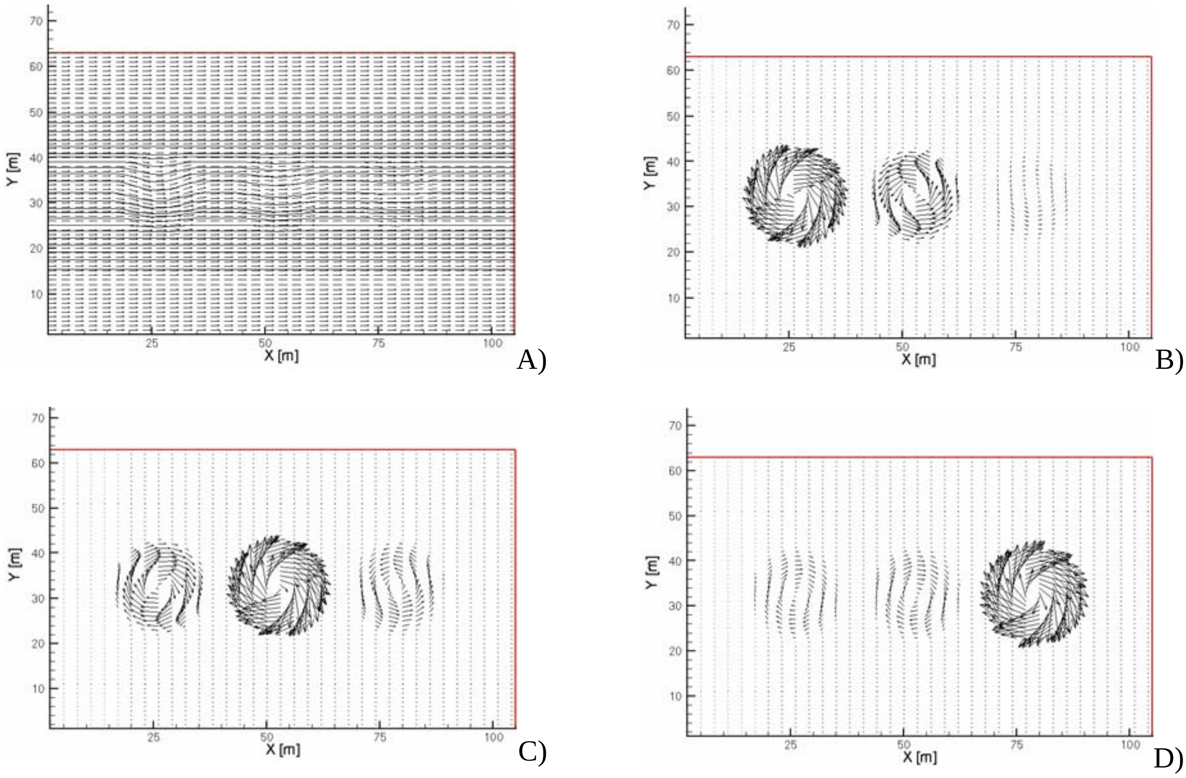

Figure 2: POD leading modes of the synthetic dataset (non-overlapping vortices). A) Mode 1 or ensemble average. B) Mode 2. C) Mode 3. D) Mode 4.

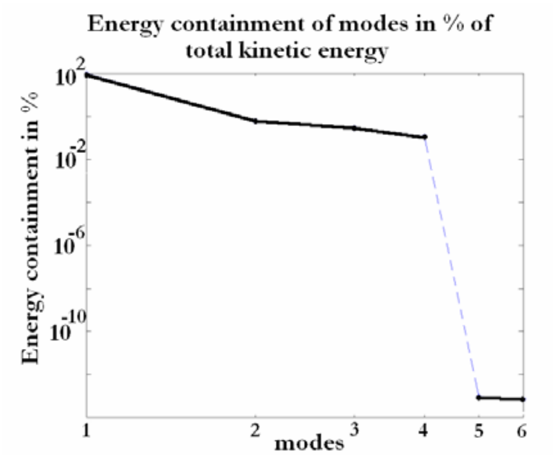

Figure 3: Mode energy contents associated to the dataset of non-overlapping vortex positions.

The direct analyse of mode 2 (**Figure 2-B**), mode 3 (**Figure 2-C**) and mode 4 (**Figure 2-D**) would lead to the following conclusion: There is a high probability to find an isolated vortex embedded in the left of the flow field, there is a lower probability to find a vortex in the centre of the field (mode 3) and finally a very low probability (but not zero) to find a vortex on the right part of the field (mode 4). Please note, that due to our choice (sign of eigenvalue) the sense of rotation of the patterns displayed in the modes is not the same as the original vortices.

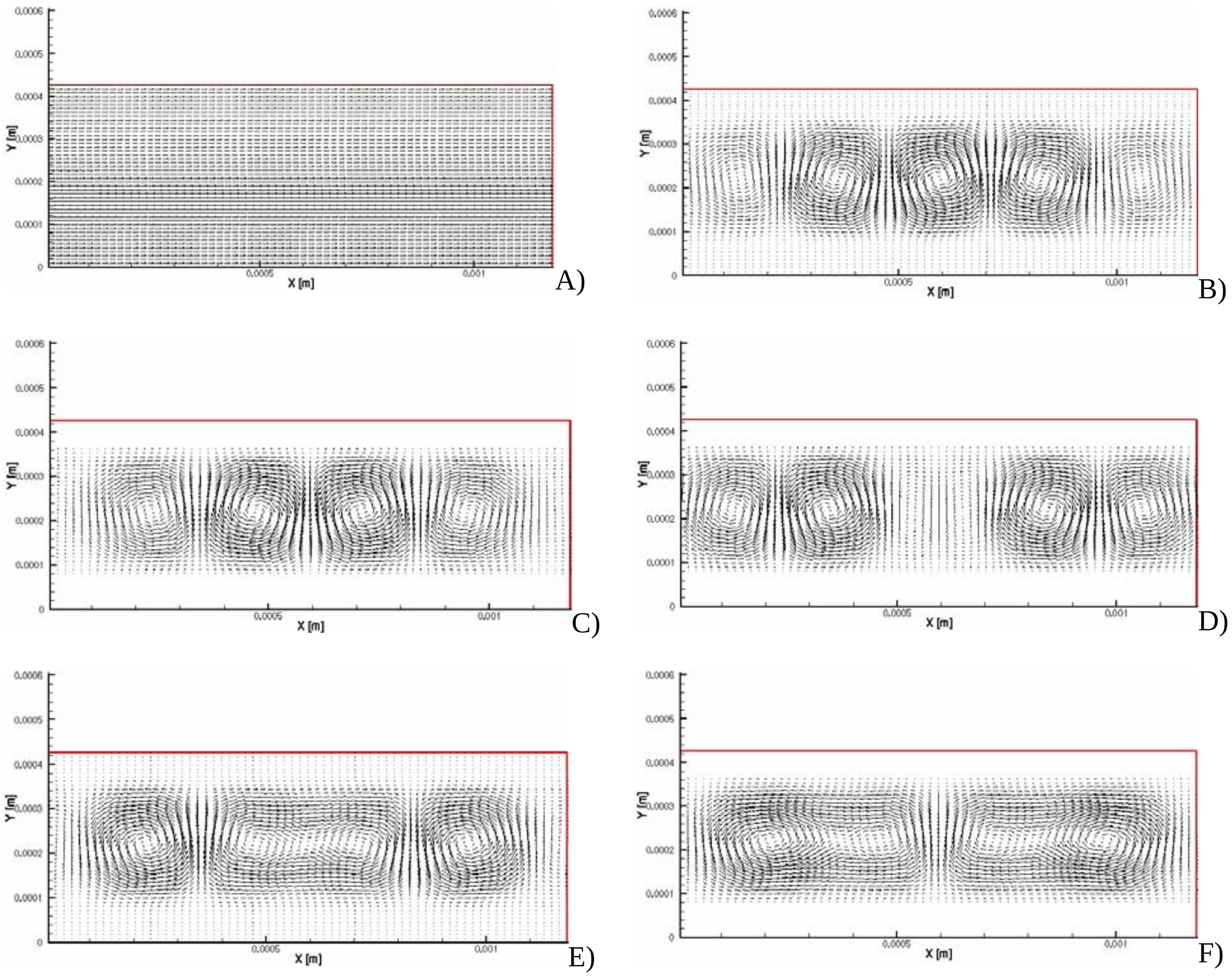

Figure 4: POD modes of vortex in overlapping positions (quasi continuous motion). A) Mode 1 or ensemble average. B) Mode 2. C) Mode 3. D) Mode 4. E) Mode 5. F) Mode 6.

The total number of meaningful modes in this case is the same as the total number of independent elements of the dataset (4 in this case). The plot of the kinetic energy containment of the modes (which is the integral of the pointwise kinetic energy computed from the vectors that can be seen in the flow field of the modes) shows clearly a significant decrease for mode 5, which means its vanishing contribution to the decomposition (**Figure 3**).

3.2 Overlapping vortex positions

The main characteristic of POD is its optimality, i.e. the number of modes needed for rebuilding any of the instantaneous (synthetic) flow fields is minimized. This test is carried out on a vortex which is moving across the domain horizontally with a step equal to 5% of its diameter (it can be handled as a quasi 'continuous' motion). Here, again, on each original picture there is only one isolated vortex in its given position and all elements of the dataset contain the vortex in different positions (equalized probability of presence). This dataset would be the one given by a camera recording the path of a unique vortex. It is noticeable that the 1st mode still coincides with the ensemble average (**Figure 4-A**). The other leading modes do not present a clear isolated vortex but a merged picture of the different fields of the initial data set. It may be noticed that several modes show similar patterns (almost pairs of vortices for modes 2, 3 & 4). In these last modes, if one extracts a vortex, it will display similar geometrical properties as the original isolated moving vortex. Unfortunately, it is not any more the case for modes 5 and 6, including already elongated structures which would mislead a direct analysis. At higher modes even smaller structures appear that were completely absent from the original dataset (**Figure 5)**. In this figure "streamlines" are used to visualize the appearance of small purely mathematical structures.

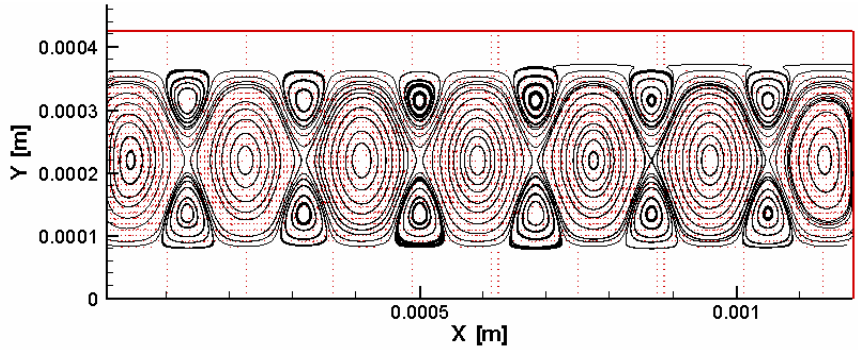

Figure 5: Mode 15 of the dataset (mainly mathematical structures)

These small structures sometimes being associated to the small scale motions would lead to an erroneous conclusion with a direct mode analysis. The energy containment of the modes (**Figure 6**) does not show the same discontinuity as it was displayed in **Figure 3**. On the contrary, it has an almost continuous distribution (**Figure 6**). The physical-like structures seem to take place on a plateau (until mode 10), the small mathematical structures begin to appear where the slope of the curve decreases.

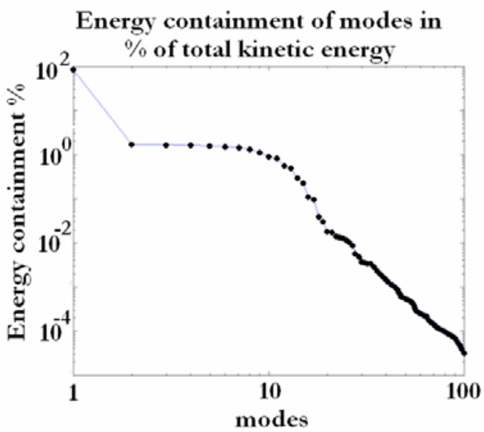

Figure 6: Energy containment of POD modes of the dataset containing continuously moving vortex

4.0 TEST OF POD ON REAL FLOW FIELD

After synthetic flow field tests, the POD algorithm was tested on a flow field over an open, rectangular cavity. The elements of the dataset were instantaneous flow fields obtained from PIV measurements. The measurements were taken in the L6 wind tunnel of the von Karman Institute for Fluid Dynamics (**Figure 7**). The Reynolds number was 4000 based on the depth of the cavity and the flow upstream could be handled as laminar **[14]**. The depth of the cavity was H=20mm with an Expansion Ratio of 1.2 and a width of wind tunnel test section of 15H.

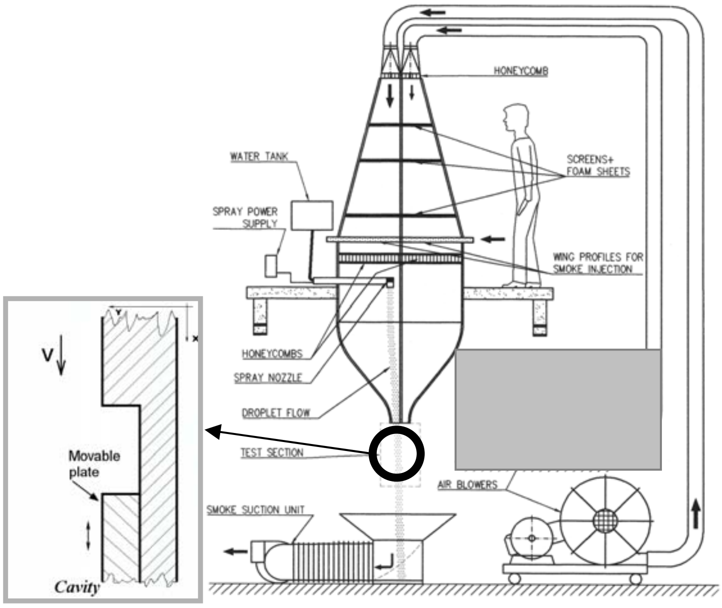

Figure 7: L6 wind tunnel and the cavity in the test section (grey box indicates non-working branch of the tunnel)

The magnification of the PCO double shutter SensyCam CCD camera at the current setup was about 74 μm/pixel with full view of the cavity. Flow realizations were captured at a frequency of 10Hz provided by the camera and the Nd-Yag laser (250mJ, 532nm). The thickness of the laser sheet was 1.1mm. The separation time was Δt = 198,4μs. Starting window size was 128x128 pixels square, 2 refinement levels were used in the multigrid option with 50% overlap of windows that resulted a final grid size of 16x16 pixels. Thus final resolution was 1.19mm in both directions. Upstream conditions were measured by hotwire and a fully laminar Blasius boundary layer was found for this Reynolds number with a thickness of 2mm at 125mm upstream from the first separation edge. Turbulence intensity based on free stream velocity was 0.9% in the whole inlet section. Results of classical statistical analysis can be found in **[14]**.

4.1 POD modes of velocity

The POD modes of instantaneous velocity can be seen in **Figure 8**. Mode 1 is the same as the mean velocity field; the 2nd and 3rd modes contain very similar vortical structures differing from each other only by a shift similarly as the isolated moving vortex of section 3.2. Thus, it is believed that the visible vortical structures are strongly connected to the movement of a representative non mathematical vortex (i.e. physical vortex).

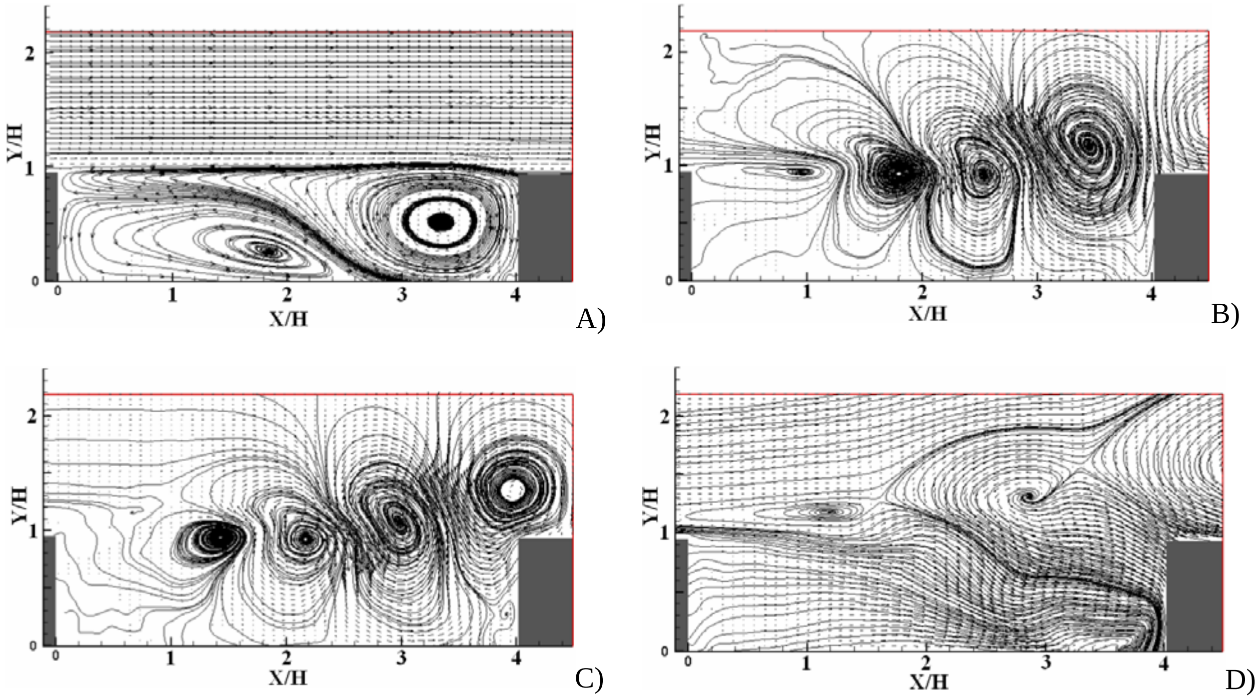

Figure 8: POD modes of velocity for the flow over a rectangular cavity at Re_H=4000. A) Mode 1. B) Mode 2. C) Mode 3. D) Mode 4.

However, present authors checked the structures contained in some instantaneous fluctuating flow fields of the velocity and similar large vortical structures were visible [14]. It may be conjectured that mode 4 is already a quite unphysical pattern. The energy containment of the POD modes can be seen in **Figure 9**. The plateau similar to the case in subsection 3.2 can be noticed containing the 2nd and 3rd modes. This shows a pair of modes that contains similar patterns (**Figure 8-B** and **C**). The 4th mode contributes with a lower energy containment, thus one can conjecture that it does not contain physical structures.

4.2 POD modes of vorticity

Beside the decomposition of velocity field, it is investigated if the vorticity field would provide meaningful decomposition leading to a better characterisation of the Coherent Structures associated to the initial fields.

Figure 10 represents the different modes of the decomposition. If, the 1st mode is always similar to the mean vorticity field, it can be only conjectured that the 2nd and 3rd modes present structures with characteristics similar to the ones obtained through the POD of velocity components (size and spatial arrangement). Nevertheless, this supplementary post-processing confirms finding of section 4.1 on the size of vortices existing in the original data set. A sure analysis of the 4th POD mode is here more delicate and will not be proposed.

Based on the vorticity distribution associated to the structures displayed on the 2nd or 3rd POD modes, the vorticity distribution of a representative vortex may be guessed. The comparison of the vorticity displayed by this representative structure and the profiles extracted at the same location (in some well chosen instantaneous field) is quite promising (**Figure 11**).

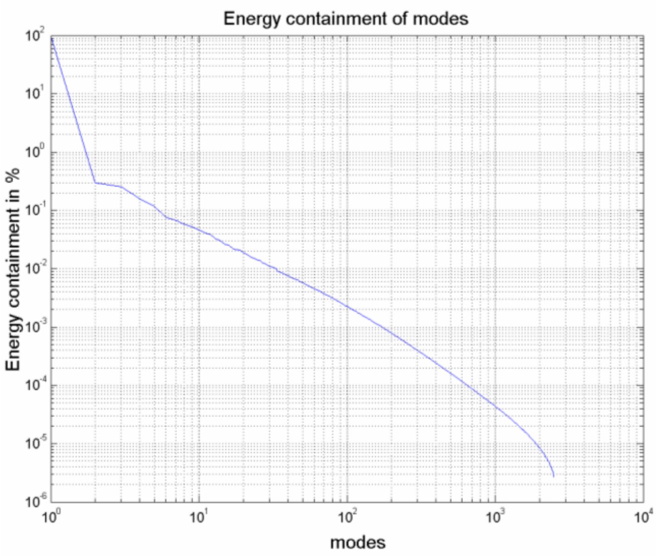

Figure 9: Energy containment of POD modes related to the total ensemble averaged kinetic energy

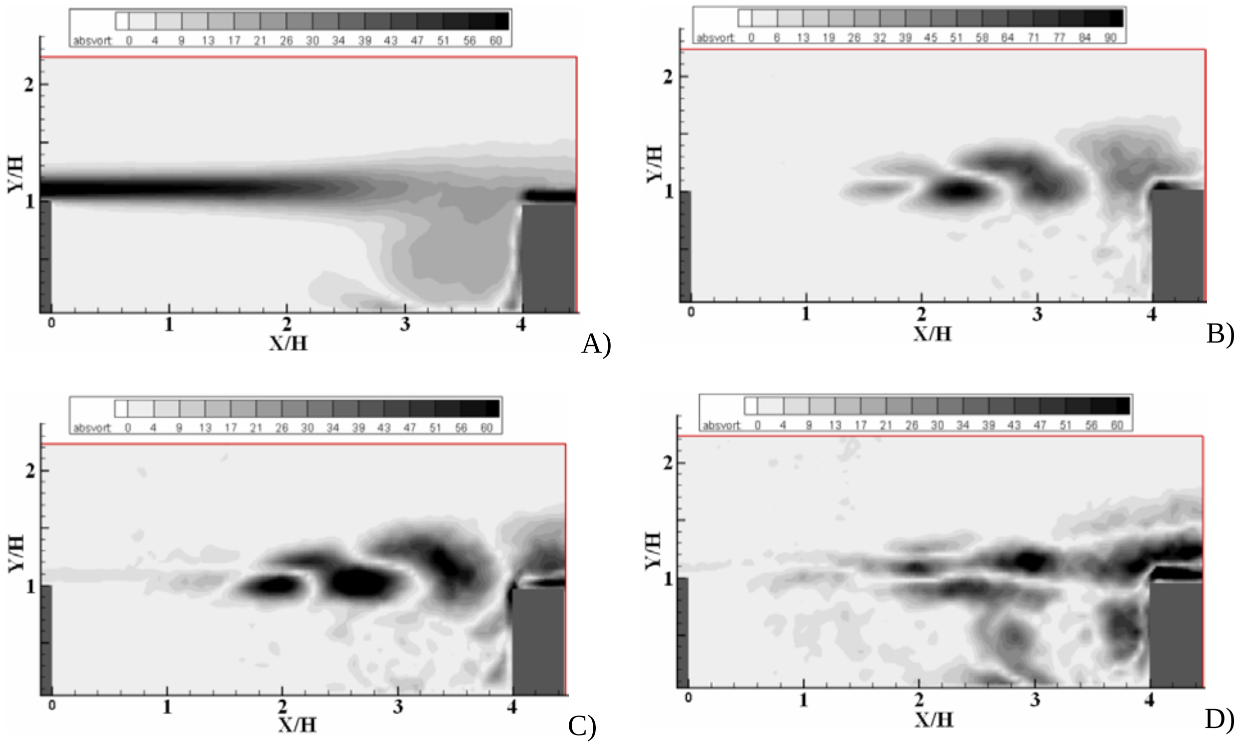

Figure 10: POD modes associated to the vorticity. A) Mode 1. B) Mode 2. C) Mode 3. D) Mode 4.

5.0 CONCLUSIONS

The purpose of this document was to apply POD on 'synthetic flow fields'. When the synthetic dataset did not contain any overlapping objects (separated in time only) then POD gave as many modes as many positions of non-overlapping objects was initially present (subsection 3.1). When overlapping positions were present then the decomposition include some mathematical structures not existing in the original synthetic dataset (subsection 3.2). These structures are seen on the modes associated with a sudden decrease of the energy containment (at the end of the plateau).

Applied to the flow over a rectangular cavity at low Reynolds number, the POD algorithm confirms the relation between the first mode and the classical ensemble average. The 2nd and 3rd modes probably showed statistically equivalent vortex-like structures passing over the mouth of the cavity. These last structures may also be extracted when the vorticity is decomposed. Positive validations were conducted to retrieve similar vortical events in the initial data set.

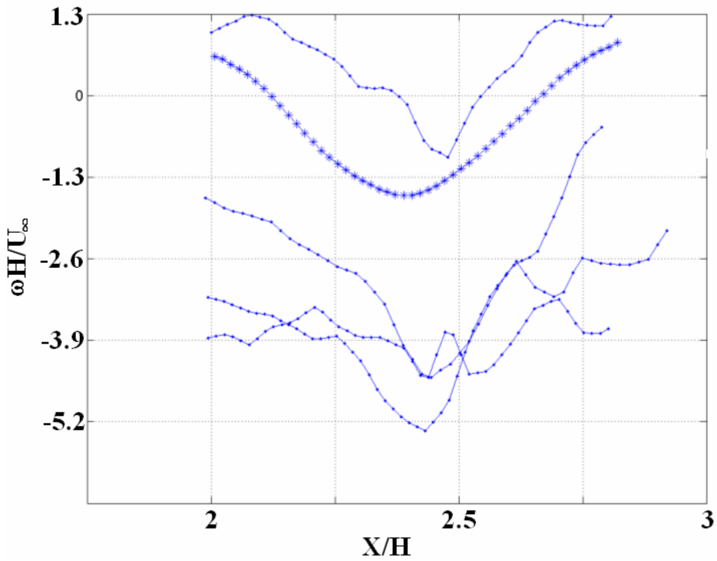

Figure 11: Vorticity (ω) profiles from instantaneous flow realizations and POD modes. Instantaneous flow field: —●—; POD field: —*—;

The validity of the present conclusion is currently tested at higher Reynolds number. Decomposition of different variables is under consideration (conditional vorticity, turbulence production tensor).

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SYMPOSIA DISCUSSION

Paper 4 "Investigation of the Link between Physics and POD Modes" presented by T. REGERT

1. Discussor's name: G. FLEMING

Q. How far can you decompose a flow field using POD and still believe the results?

R. If we decompose **u** (**r**, t) by POD, then the 1st mode is identified to the mean flow field **U** (**r**). The 2nd and 3rd modes contain the most energetic structures which are surimposed onto the mean flow field. This way, by POD, we obtain not only the magnitude of "fluctuations", but also their structure. As fluctuations are mainly caused by vortices, POD extracts them. The modes above the 3rd mode are already not reliable. They might contain physical structures but high experience of the researcher is needed for that. Basically, POD is not appropriate to study decaying or homogened turbulence, but for studying flows which are dominated by the effects of large scale, energetic vortices, caused by BL separate.